

A circle is the set of all points in a plane that are a given distance from a given point in the plane called the center. Various parts of a circle are labeled in the figure at the right.

The distance around a circle is called the circumference.

Circumference of a Circle	If a circle has a circumference of C units and a radius of r units, then $C = 2\pi r$.
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Example: Find the circumference of the circle shown at the right.

$$\begin{aligned} C &= 2\pi r \\ C &= 2\pi(13) \\ C &= 26\pi \\ C &\approx 81.7 \end{aligned}$$

The circumference is about 81.7 cm.

Refer to $\odot S$ for Exercises 1-6.

1. Name the center of $\odot S$. **S**
2. Name three radii of $\odot S$. **SM, ST, SR**
3. Name a diameter. **RT**
4. Name a chord. **XY or ET**

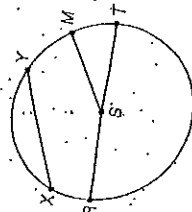
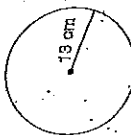
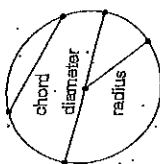
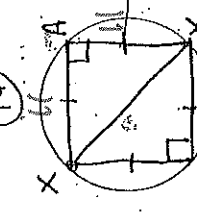
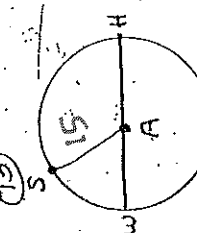
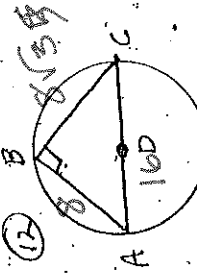
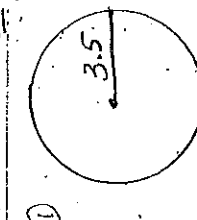
5. If $RT = 8.2$, find SM . **4.1**

6. Is $\overline{SR} \cong \overline{SM}$? Explain. **yes both radii**

In Exercises 7-10, the radius, diameter, or circumference of a circle is given. Find the other measures to the nearest tenth.

7. $r = 7$, $d = 14$, $C = 44$ 8. $d = 32.4$, $r = 16.2$, $C = 101.8$
9. $C = 116.5$, $d = 37.1$, $r = 18.6$ 10. $r = 12$, $d = 24$, $C = 75.4$

Find the exact circumference.



An angle whose vertex is at the center of a circle is called a **central angle**. A central angle separates a circle into two arcs called a **major arc** and a **minor arc**. In the circle at the right, $\angle CEF$ is a central angle. Points C and F and all points of the circle interior to $\angle CEF$ form a minor arc called arc CF . This is written \widehat{CF} . Points C and F and all points of the circle exterior to $\angle CEF$ form a major arc called \widehat{CEF} .

You can use central angles to find both the degree measure and the length of an arc. The arcs determined by a diameter are called **semicircles** and have measures of 180.

Examples: In $\odot R$, $m\angle ARB = 42$, $RB = 12$, and \widehat{AC} is a diameter.

1. Find $m\widehat{AB}$ and $m\widehat{ACB}$.
Since $\angle ARB$ is a central angle and $m\angle ARB = 42$, then $m\widehat{AB} = 42$.
 $m\widehat{ACB} = 360 - m\widehat{AB} = 360 - 42$ or 318
2. Find the length of \widehat{AB} .
First, find what part of the circle is represented by $\angle ARB$.

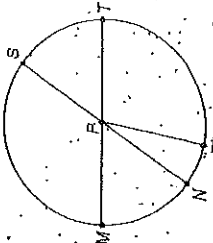
$$\frac{42}{360} = \frac{r}{60}$$

So, the length of \widehat{AB} is $\frac{7}{60}$ of the circumference of $\odot R$.
length of $\widehat{AB} = \frac{7}{60}(2\pi r)$

$$= \frac{7}{60}(2\pi)(12) \text{ or about } 8.8 \text{ units}$$

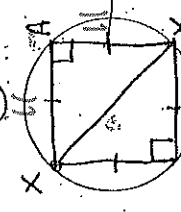
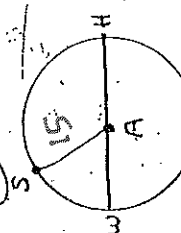
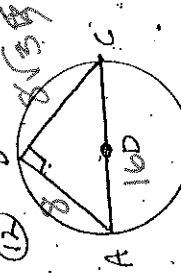
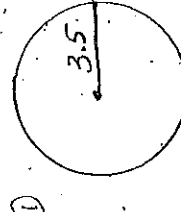
Refer to $\odot P$ for Exercises 1-8. If \widehat{SN} and \widehat{MT} are diameters with $m\angle SPT = 51$ and $m\angle NPR = 29$, determine whether each arc is a minor arc, a major arc, or a semicircle. Then find the degree measure of each arc.

1. $m\widehat{NR}$ minor, 29°
 2. $m\widehat{ST}$ minor, 51°
 3. $m\widehat{TSR}$ major, 260°
 4. $m\widehat{MST}$ 180°
- If $MT = 15$, find the length of each arc. Round to the nearest tenth.
5. \widehat{NR} 3.8
 6. \widehat{ST} 6.7
 7. \widehat{TSR} 34
 8. \widehat{MST} 23.6



⑨ Find the area of the sector contained $\angle NPR$

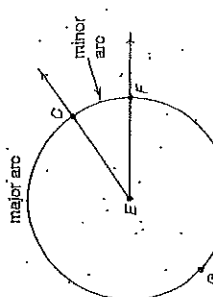
$$\frac{29}{360} \cdot \pi (7.5)^2 = 14.2$$



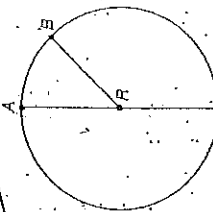
$$C = 2\pi(3.5) = 7\pi$$

$$\text{radius} = 8 \quad BC = ? \quad AB = 8 \quad 8(3\pi)$$

$$SA = 15 \quad AH = C = ? \quad 15 \cdot 30\pi$$



Area of sector
$$\frac{CA}{360} \cdot \pi r^2$$



arc length =
$$\frac{CA}{360} \cdot \pi d$$

The following theorems state relationships between arcs, chords, and diameters.

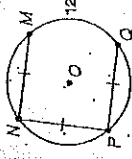
- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example: In the circle, O is the center, $OD = 15$, and $CD = 24$. Find x .

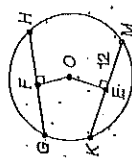
$$\begin{aligned} OD &= \frac{1}{2} CD \\ 15 &= \frac{1}{2} (24) \\ &= 12 \\ (OE)^2 + (ED)^2 &= (OD)^2 \\ x^2 + 12^2 &= 15^2 \\ x^2 + 144 &= 225 \\ x^2 &= 81 \\ x &= 9 \end{aligned}$$

In each circle, O is the center. Find each measure.

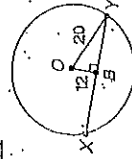
1. $m\widehat{NP}$



2. KM



3. \widehat{XY}



4. Suppose a chord is 20 inches long and is 24 inches from the center of the circle. Find the length of the radius.

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5. Suppose a chord of a circle is 5 inches from the center and is 24 inches long. Find the length of the radius.

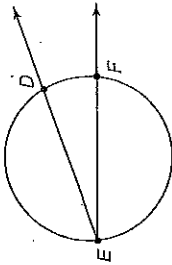
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6. Suppose the diameter of a circle is 30 centimeters long and a chord is 24 centimeters long. Find the distance between the chord and the center of the circle.

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Inscribed Angles

An inscribed angle of a circle is an angle whose vertex is on the circle and whose sides contain chords of the circle. We say that $\angle DEF$ intercepts \widehat{DF} . The following theorems involve inscribed angles.



- If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.
- If two inscribed angles of a circle or congruent circles intercept congruent arcs or the same arc, then the angles are congruent.
- If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

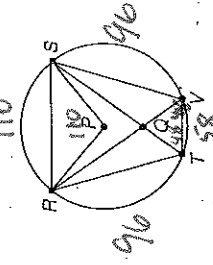
Example: In the circle above, find $m\angle DEF$ if $m\widehat{DF} = 28$. Since $\angle DEF$ is an inscribed angle, $m\angle DEF = \frac{1}{2} m\widehat{DF} = \frac{1}{2} (28) = 14$.

$$\begin{array}{r} 110 \\ 360 \\ 96 \\ 96 \\ \hline 302 \\ 58 \end{array}$$

In $\odot P$,

1. Name the intercepted arc for $\angle RTS$.

RS



2. Name an inscribed angle. $\angle RVS$

3. Name a central angle. $\angle RPS$

In $\odot P$, $m\widehat{SV} = 96$ and $m\angle RPS = 110$. Find each measure. Also $m\angle RVT = 48$.

4. $m\angle RVS$

5. $m\widehat{RS}$

6. $m\angle RTS$

7. $m\widehat{VT}$

8. $m\angle RV$

9. $m\angle TVQ$

10. $m\angle RVT$

11. $m\angle RVS$

12. $m\angle RTS$

13. $m\angle RVT$

14. $m\angle RVS$

15. $m\angle RTS$

16. $m\angle RVT$

17. $m\angle RVS$

18. $m\angle RTS$

19. $m\angle RVT$

20. $m\angle RVS$

21. $m\angle RTS$

22. $m\angle RVT$

23. $m\angle RVS$

24. $m\angle RTS$

25. $m\angle RVT$

26. $m\angle RVS$

27. $m\angle RTS$

28. $m\angle RVT$

29. $m\angle RVS$

30. $m\angle RTS$

31. $m\angle RVT$

32. $m\angle RVS$

33. $m\angle RTS$

34. $m\angle RVT$

35. $m\angle RVS$

36. $m\angle RTS$

37. $m\angle RVT$

38. $m\angle RVS$

39. $m\angle RTS$

40. $m\angle RVT$

41. $m\angle RVS$

42. $m\angle RTS$

43. $m\angle RVT$

44. $m\angle RVS$

45. $m\angle RTS$

46. $m\angle RVT$

47. $m\angle RVS$

48. $m\angle RTS$

49. $m\angle RVT$

50. $m\angle RVS$

51. $m\angle RTS$

52. $m\angle RVT$

53. $m\angle RVS$

54. $m\angle RTS$

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60. $m\angle RTS$

61. $m\angle RVT$

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63. $m\angle RTS$

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73. $m\angle RVT$

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79. $m\angle RVT$

80. $m\angle RVS$

81. $m\angle RTS$

82. $m\angle RVT$

83. $m\angle RVS$

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94. $m\angle RVT$

95. $m\angle RVS$

96. $m\angle RTS$

97. $m\angle RVT$

98. $m\angle RVS$

99. $m\angle RTS$

100. $m\angle RVT$